The Basic Primitive Recursive Functions are Lambda Representable

Lemma lam.1. The functions 0, S, and \( P^n \) are lambda representable.

Proof. Zero, \( \mathbb{0} \), is just \( \lambda x. \lambda y. y \).

The successor function \( S \), is defined by \( S(u) = \lambda x. \lambda y. x(uxy) \). You should think about why this works; for each numeral \( n \), thought of as an iterator, and each function \( f \), \( S(n, f) \) is a function that, on input \( y \), applies \( f \) \( n \) times starting with \( y \), and then applies it once more.

There is nothing to say about projections: \( P^i(x_0, \ldots, x_{n-1}) = x_i \). In other words, by our conventions, \( P^i \) is the lambda term \( \lambda x_0. \ldots \lambda x_{n-1}. x_i \). \qed

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Bibliography