lam.1 The Basic Primitive Recursive Functions are Lambda Representable

cmp:lam:bas:sec

Lemma lam.1. *The functions* 0*,* S*, and* P_i^n *are lambda representable.*

Proof. Zero, $\overline{0}$, is just $\lambda x.\, \lambda y.\, y$.

The successor function $\overline{S}$, is defined by $\overline{S}(u) = \lambda x.\, \lambda y.\, x(uxy)$. You should think about why this works; for each numeral $\overline{n}$, thought of as an iterator, and each function f, $S(\overline{n}, f)$ is a function that, on input y, applies f n times starting with y, and then applies it once more.

There is nothing to say about projections: $\overline{P_i^n}(x_0, \ldots, x_{n-1}) = x_i$. In other words, by our conventions, $\overline{P_i^n}$ is the lambda term $\lambda x_0.\, \ldots \lambda x_{n-1}.\, x_i$. □

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Bibliography