

## lam.1 The Basic Primitive Recursive Functions are Lambda Representable

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**Lemma lam.1.** *The functions  $0$ ,  $S$ , and  $P_i^n$  are lambda representable.*

*Proof.* Zero,  $\bar{0}$ , is just  $\lambda x. \lambda y. y$ .

The successor function  $\bar{S}$ , is defined by  $\bar{S}(u) = \lambda x. \lambda y. x(uxy)$ . You should think about why this works; for each numeral  $\bar{n}$ , thought of as an iterator, and each function  $f$ ,  $S(\bar{n}, f)$  is a function that, on input  $y$ , applies  $f$   $n$  times starting with  $y$ , and then applies it once more.

There is nothing to say about projections:  $\bar{P}_i^n(x_0, \dots, x_{n-1}) = x_i$ . In other words, by our conventions,  $\bar{P}_i^n$  is the lambda term  $\lambda x_0. \dots \lambda x_{n-1}. x_i$ .  $\square$

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### Bibliography