Theorem thy.1. There is a universal partial computable function \( Un(k, x) \). In other words, there is a function \( Un(k, x) \) such that:

1. \( Un(k, x) \) is partial computable.
2. If \( f(x) \) is any partial computable function, then there is a natural number \( k \) such that \( f(x) \simeq Un(k, x) \) for every \( x \).

Proof. Let \( Un(k, x) \simeq U(\mu s T(k, x, s)) \) in Kleene’s normal form theorem. 

This is just a precise way of saying that we have an effective enumeration of the partial computable functions; the idea is that if we write \( f_k \) for the function defined by \( f_k(x) = Un(k, x) \), then the sequence \( f_0, f_1, f_2, \ldots \) includes all the partial computable functions, with the property that \( f_k(x) \) can be computed “uniformly” in \( k \) and \( x \). For simplicity, we are using a binary function that is universal for unary functions, but by coding sequences of numbers we can easily generalize this to more arguments. For example, note that if \( f(x, y, z) \) is a 3-place partial recursive function, then the function \( g(x) \simeq f((x)_0, (x)_1, (x)_2) \) is a unary recursive function.