

thy.1 The Universal Partial Computable Function

cmp:thy:uni:
sec

cmp:thy:uni:
thm:univ-comp

Theorem thy.1. *There is a universal partial computable function $\text{Un}(k, x)$. In other words, there is a function $\text{Un}(k, x)$ such that:*

1. $\text{Un}(k, x)$ is partial computable.
2. If $f(x)$ is any partial computable function, then there is a natural number k such that $f(x) \simeq \text{Un}(k, x)$ for every x .

Proof. Let $\text{Un}(k, x) \simeq U(\mu s T(k, x, s))$ in Kleene's normal form theorem. \square

This is just a precise way of saying that we have an effective enumeration of the partial computable functions; the idea is that if we write f_k for the function defined by $f_k(x) = \text{Un}(k, x)$, then the sequence f_0, f_1, f_2, \dots includes all the partial computable functions, with the property that $f_k(x)$ can be computed "uniformly" in k and x . For simplicity, we are using a binary function that is universal for unary functions, but by coding sequences of numbers we can easily generalize this to more arguments. For example, note that if $f(x, y, z)$ is a 3-place partial recursive function, then the function $g(x) \simeq f((x)_0, (x)_1, (x)_2)$ is a unary recursive function. [explanation](#)

Photo Credits

Bibliography