cmp:thy:uni:sec

# thy.1 The Universal Partial Computable Function

cmp:thy:uni:thm:univ-comp

**Theorem thy.1.** *There is a universal partial computable function* $\mathrm{Un}(k, x)$*. In other words, there is a function* $\mathrm{Un}(k, x)$ *such that:*

1. $\mathrm{Un}(k, x)$ *is partial computable.*
2. *If* $f(x)$ *is any partial computable function, then there is a natural number* $k$ *such that* $f(x) \simeq \mathrm{Un}(k, x)$ *for every* $x$*.*

*Proof.* Let $\mathrm{Un}(k, x) \simeq U(\mu s\, T(k, x, s))$ in Kleene's normal form theorem. □

This is just a precise way of saying that we have an effective enumeration of the partial computable functions; the idea is that if we write $f_k$ for the function defined by $f_k(x) = \mathrm{Un}(k, x)$, then the sequence $f_0$, $f_1$, $f_2$, . . . includes all the partial computable functions, with the property that $f_k(x)$ can be computed "uniformly" in $k$ and $x$. For simplicity, we am using a binary function that is universal for unary functions, but by coding sequences of numbers we can easily generalize this to more arguments. For example, note that if $f(x, y, z)$ is a 3-place partial recursive function, then the function $g(x) \simeq f((x)_0, (x)_1, (x)_2)$ is a unary recursive function. explanation

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## Bibliography