thy.1 The s-m-n Theorem

cmp:thy:smn:sec The next theorem is known as the "s-m-n theorem," for a reason that will be clear in a moment. The hard part is understanding just what the theorem says; once you understand the statement, it will seem fairly obvious. explanation

cmp:thy:smn:thm:s-m-n **Theorem thy.1.** *For each pair of natural numbers n and m, there is a primitive recursive function s^m_n such that for every sequence x, a_0, …, a_{m-1}, y_0 ,…, y_{n-1}, we have*

$$\varphi^n_{s^m_n(x,a_0,\dots,a_{m-1})}(y_0,\dots,y_{n-1}) \simeq \varphi^{m+n}_x(a_0,\dots,a_{m-1},y_0,\dots,y_{n-1}).$$

It is helpful to think of s^m_n as acting on *programs*. That is, s^m_n takes a program, x, for an $(m+n)$-ary function, as well as fixed inputs a_0, …, a_{m-1}; and it returns a program, $s^m_n(x,a_0,\dots,a_{m-1})$, for the n-ary function of the remaining arguments. It you think of x as the description of a Turing machine, then $s^m_n(x,a_0,\dots,a_{m-1})$ is the Turing machine that, on input y_0, …, y_{n-1}, prepends a_0, …, a_{m-1} to the input string, and runs x. Each s^m_n is then just a primitive recursive function that finds a code for the appropriate Turing machine. explanation

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Bibliography