

## thy.1 Computable Sets

[cmp:thy:cps:sec](#) We can extend the notion of computability from computable functions to computable sets:

**Definition thy.1.** Let  $S$  be a set of natural numbers. Then  $S$  is *computable* iff its characteristic function is. In other words,  $S$  is computable iff the function

$$\chi_S(x) = \begin{cases} 1 & \text{if } x \in S \\ 0 & \text{otherwise} \end{cases}$$

is computable. Similarly, a relation  $R(x_0, \dots, x_{k-1})$  is computable if and only if its characteristic function is.

Computable sets are also called *decidable*.

[explanation](#)

Notice that we now have a number of notions of computability: for partial functions, for functions, and for sets. Do not get them confused! The Turing machine computing a partial function returns the output of the function, for input values at which the function is defined; the Turing machine computing a set returns either 1 or 0, after deciding whether or not the input value is in the set or not.

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## Bibliography