If $\prec$ *is ...*	*then ... is true in* $\mathfrak{M}$*:*
transitive: $\forall u \forall v \forall w((u \prec v \wedge v \prec w) \rightarrow u \prec w)$	$\mathsf{FF}p \rightarrow \mathsf{F}p$
linear: $\forall w \forall v(w \prec v \vee w = v \vee v \prec w)$	$(\mathsf{FP}p \vee \mathsf{PF}p) \rightarrow (\mathsf{P}p \vee p \vee \mathsf{F}p)$
dense: $\forall w \forall v(w \prec v \rightarrow \exists u(w \prec u \wedge u \prec v))$	$\mathsf{F}p \rightarrow \mathsf{FF}p$
unbounded (past): $\forall w \exists v(v \prec w)$	$\mathsf{H}p \rightarrow \mathsf{P}p$
unbounded (future): $\forall w \exists v(w \prec v)$	$\mathsf{G}p \rightarrow \mathsf{F}p$

Table 1: Some temporal frame correspondence properties.

aml:tl:acc:
tab:correspondence

tl.1 Properties of Temporal Frames

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sec

Given that our temporal models do not impose any conditions on the relation $\prec$, the only one of our familiar axioms that holds in all models is K, or its analogues K_{G} and K_{H}:

$$\mathsf{G}(p \rightarrow q) \rightarrow (\mathsf{G}p \rightarrow \mathsf{G}q) \qquad (K_{\mathsf{G}})$$

$$\mathsf{H}(p \rightarrow q) \rightarrow (\mathsf{H}p \rightarrow \mathsf{H}q) \qquad (K_{\mathsf{H}})$$

However, if we want our models to impose stricter conditions on how time is represented, for instance by ensuring that $\prec$ is a linear order, then we will end up with other validities in our models.

Several of the properties from Table 1 might seem like desirable features for a model that is intended to represent time. However, it is worth noting that, even though we can impose whichever conditions we like on the $\prec$ relation, not all conditions correspond to formulas that can be expressed in the language of temporal logic. For example, irreflexivity, or the idea that $\forall w \neg(w \prec w)$, does not have a corresponding formula in temporal logic.

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Bibliography