tl.1 Possible Histories

aml:tl:poss:sec

The relational models of temporal logic that we have been using are extremely flexible, since we do not have to place any restrictions on the accessibility relation. This means that temporal models can branch in the past and in the future, but we might want to consider a more "modal" conception of branching, in which we consider sequences of events as possible histories. This does not necessarily require changing our language, though we might also add our "ordinary" modal operators $\Box$ and $\Diamond$, and we could also consider adding epistemic accessibility relations to represent changes in agents' knowledge over time.

Definition tl.1. A *possible histories model* for the temporal language is a triple $\mathfrak{M} = \langle T, C, V \rangle$, where

1. T is a nonempty set, interpreted as states in time.
2. C is a set of computational paths, or *possible histories* of a system. In other words, C is a set of sequences σ of states s_1, s_2, s_3, …, where every $s_i \in T$.
3. V is a function assigning to each propositional variable p a set $V(p)$ of points in time.

To make things simpler, we will also generally assume that when a history is in C, then so are all of its suffixes. For example, if s_1, s_2, s_3 is a sequence in C, then so are s_2, s_3 and s_3. Also, when two states s_i and s_j appear in a sequence σ, we say that $s_i \prec_\sigma s_j$ when $i < j$. When $t \in V(p)$ we say p is *true at* t.

The one relevant change is that when we evaluate the truth of a formula at a point in time t in a model $\mathfrak{M}$, we do so relative to a history σ, in which t appears as a state. We do not need to change any of the semantics for propositional variables or for truth-functional connectives, though. All of those are exactly as they were in **??**, since none of those will make reference to σ. However, we now redefine our future operator F and add our $\Diamond$ operator with respect to these histories.

aml:tl:poss:defn:phmodels

Definition tl.2. *Truth of a formula* φ *at* t, σ in $\mathfrak{M}$, in symbols: $\mathfrak{M}, t, \sigma \Vdash \varphi$:

aml:tl:poss:defn:sub:phmodels-f

1. $\varphi \equiv \mathsf{F}\psi$: $\mathfrak{M}, t, \sigma \Vdash \varphi$ iff $\mathfrak{M}, t', \sigma \Vdash \psi$ for some $t' \in T$ such that $t \prec_\sigma t'$.

aml:tl:poss:defn:sub:phmodels-diamond

2. $\varphi \equiv \Diamond\psi$: $\mathfrak{M}, t, \sigma \Vdash \varphi$ iff $\mathfrak{M}, t, \sigma' \Vdash \psi$ for some $\sigma' \in C$ in which t occurs.

Other temporal and modal operators can be defined similarly. However, we can now represent claims that combine tense and modality. For example, we might symbolize "p will not occur, but it might have occurred" using the formula $\neg\mathsf{F}p \wedge \Diamond\mathsf{F}p$. This would hold at a point and a history at which p does not become true at a successor state, but there is an alternative history at which p will become true.

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Bibliography