tl.1 Additional Operators for Temporal Logic

aml:tl:ext: sec

In addition to the unary operators for past and future, temporal logics also sometimes include binary operators S and U, intended to symbolize "since" and "until". This means adding S and U into the language of temporal logic and adding the following clause into the definition of a temporal formula:

> If φ and ψ are formulas, then $(\mathsf{S}\varphi\psi)$ and $(\mathsf{U}\varphi\psi)$ are both formulas.

The semantics for these operators are then given as follows:

aml:tl:ext: defn:since-until

Definition tl.1. *Truth of a formula φ at t* in a $\mathfrak{M}$:

aml:tl:ext: defn:sub:mmodels-since

1. $\varphi \equiv \mathsf{S}\psi\chi$: $\mathfrak{M}, t \Vdash \varphi$ iff $\mathfrak{M}, t' \Vdash \psi$ for some $t' \in T$ with $t' \prec t$, and for all s with $t' \prec s \prec t$, $\mathfrak{M}, s \Vdash \chi$

aml:tl:ext: defn:sub:mmodels-until

2. $\varphi \equiv \mathsf{U}\psi\chi$: $\mathfrak{M}, t \Vdash \varphi$ iff $\mathfrak{M}, t' \Vdash \psi$ for some $t' \in T$ with $t \prec t'$, and for all s with $t \prec s \prec t'$, $\mathfrak{M}, s \Vdash \chi$

The intuitive reading of $\mathsf{S}\psi\chi$ is "Since ψ was the case, χ has been the case." And the intuitive reading of $\mathsf{U}\psi\chi$ is "Until ψ will be the case, χ will be the case."

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Bibliography