el.1 Semantics of Public Announcement Logic

aml:el:psm:sec Relational models for public announcement logics are the same as they were in epistemic logics. However, the semantics for the public announcement operator are something new.

aml:el:psm:defn:mmodels **Definition el.1.** *Truth of a formula* φ *at* w in a $\mathfrak{M} = \langle W, R, V \rangle$, in symbols: $\mathfrak{M}, w \Vdash \varphi$, is defined inductively as follows:

1. $\varphi \equiv \bot$: Never $\mathfrak{M}, w \Vdash \bot$.
2. $\varphi \equiv \top$: Always $\mathfrak{M}, w \Vdash \top$.
3. $\mathfrak{M}, w \Vdash p$ iff $w \in V(p)$
4. $\varphi \equiv \neg\psi$: $\mathfrak{M}, w \Vdash \varphi$ iff $\mathfrak{M}, w \nVdash \psi$.
5. $\varphi \equiv (\psi \wedge \chi)$: $\mathfrak{M}, w \Vdash \varphi$ iff $\mathfrak{M}, w \Vdash \psi$ and $\mathfrak{M}, w \Vdash \chi$.
6. $\varphi \equiv (\psi \vee \chi)$: $\mathfrak{M}, w \Vdash \varphi$ iff $\mathfrak{M}, w \Vdash \psi$ or $\mathfrak{M}, w \Vdash \chi$ (or both).
7. $\varphi \equiv (\psi \rightarrow \chi)$: $\mathfrak{M}, w \Vdash \varphi$ iff $\mathfrak{M}, w \nVdash \psi$ or $\mathfrak{M}, w \Vdash \chi$.
8. $\varphi \equiv (\psi \leftrightarrow \chi)$: $\mathfrak{M}, w \Vdash \varphi$ iff either both $\mathfrak{M}, w \Vdash \psi$ and $\mathfrak{M}, w \Vdash \chi$ or neither $\mathfrak{M}, w \Vdash \psi$ nor $\mathfrak{M}, w \Vdash \chi$.
9. aml:el:psm:defn:sub:mmodels-box $\varphi \equiv \mathsf{K}_a\psi$: $\mathfrak{M}, w \Vdash \varphi$ iff $\mathfrak{M}, w' \Vdash \psi$ for all $w' \in W$ with $R_a ww'$
10. aml:el:psm:defn:sub:mmodels-pal $\varphi \equiv [\psi]\chi$: $\mathfrak{M}, w \Vdash \varphi$ iff $\mathfrak{M}, w \Vdash \psi$ implies $\mathfrak{M} \mid \psi, w \Vdash \chi$

    Where $\mathfrak{M} \mid \psi = \langle W', R', V' \rangle$ is defined as follows:

    a) $W' = \{u \in W : \mathfrak{M}, u \Vdash \psi\}$. So the worlds of $\mathfrak{M} \mid \psi$ are the worlds in $\mathfrak{M}$ at which ψ holds.
    b) $R'_a = R_a \cap (W' \times W')$. Each agent's accessibility relation is simply restricted to the worlds that remain in W'.
    c) $V'(p) = \{u \in W' : u \in V(p)\}$. Similarly, the propositional valuations at worlds remain the same, representing the idea that informational events will not change the truth value of propositional variables.

What is distinctive, then, about public announcement logics, is that the truth of a formula at $\mathfrak{M}$ can sometimes only be decided by referring to a model other than $\mathfrak{M}$ itself.

Notice also that our semantics treats the announcement operator as a $\Box$ operator, and so if a formula φ cannot be truthfully announced at a world, then $[\varphi]B$ will hold there trivially, just as all $\Box$ formulas hold at endpoints.

We can see the public announcement of a formula as shrinking a model, or restricting it to the worlds at which the formula was true. Figure 1 gives an example of the effects of publicly announcing p. One notable thing about

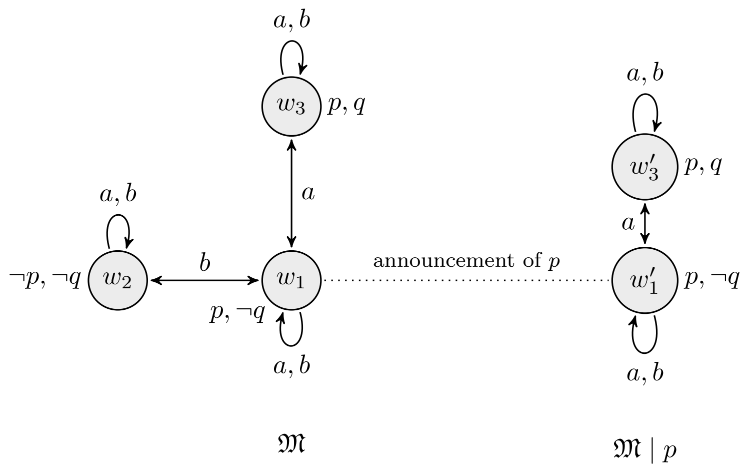

Figure 1: Before and after the public announcement of p.

aml:el:psm: fig:announcement-example

that model is that agent b learns that p as a result of the announcement, while agent a does not (since a already knew that p was true).

More formally, we have $\mathfrak{M}, w_1 \Vdash \neg \mathsf{K}_b p$ but $\mathfrak{M} \mid p, w_1' \Vdash \mathsf{K}_b p$. This implies that $\mathfrak{M}, w_1 \Vdash [p]\mathsf{K}_b p$. But we have some even stronger claims that we can make about the result of the announcement. In fact, it is the case that $\mathfrak{M}, w_1 \Vdash [p]\mathsf{C}_{\{a,b\}} p$. In other words, after p is announced, it becomes *common knowledge*.

We might wonder, though, whether this holds in the general case, and whether a truthful announcement of φ will *always* result in φ becoming common knowledge. It may be surprising that the answer is no. And in fact, it is possible to truthfully announce formulas that will no longer be true once they are announced. For example, consider the effects of announcing $p \land \neg \mathsf{K}_b p$ at w_1 in Figure 1. In fact, $\mathfrak{M} \mid p$ and $\mathfrak{M} \mid (p \land \neg \mathsf{K}_b p)$ are the same model. However, as we have already noted, $\mathfrak{M} \mid p, w_1' \Vdash \mathsf{K}_b p$. Therefore, $\mathfrak{M} \mid (p \land \neg \mathsf{K}_b p), w_1' \Vdash \neg(p \land \neg \mathsf{K}_b p)$, so this is a formula that becomes false once it has been announced.

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Bibliography