

el.1 The Language of Epistemic Logic

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Definition el.1. Let G be a set of agent-symbols. The basic language of multi-agent epistemic logic contains

1. The propositional constant for **falsity** \perp .
2. The propositional constant for **truth** \top .
3. A **denumerable** set of **propositional variables**: p_0, p_1, p_2, \dots
4. The propositional connectives: \neg (negation), \wedge (conjunction), \vee (disjunction), \rightarrow (**conditional**), \leftrightarrow (**biconditional**).
5. The knowledge operator K_a where $a \in G$.

If we are only concerned with the knowledge of a single agent in our system, we can drop the reference to the set G , and individual agents. In that case, we only have the basic operator K .

Definition el.2. *Formulas* of the epistemic language are inductively defined as follows:

1. \perp is an atomic **formula**.
2. \top is an atomic **formula**.
3. Every propositional variable p_i is an (atomic) **formula**.
4. If φ is a **formula**, then $\neg\varphi$ is a **formula**.
5. If φ and ψ are **formulas**, then $(\varphi \wedge \psi)$ is a **formula**.
6. If φ and ψ are **formulas**, then $(\varphi \vee \psi)$ is a **formula**.
7. If φ and ψ are **formulas**, then $(\varphi \rightarrow \psi)$ is a **formula**.
8. If φ and ψ are **formulas**, then $(\varphi \leftrightarrow \psi)$ is a **formula**.
9. If φ is a **formula** and $a \in G$, then $K_a\varphi$ is a **formula**.
10. Nothing else is a **formula**.

If a **formula** φ does not contain K_a , we say it is *modal-free*.

Definition el.3. While the K operator is intended to symbolize individual knowledge, E , often read as “everybody knows,” symbolizes group knowledge. Where $G' \subseteq G$, we define $E_{G'}\varphi$ as an abbreviation for

$$\bigwedge_{b \in G'} K_b \varphi.$$

We can also define an even stronger sense of knowledge, namely *common knowledge* among a group of agents G . When a piece of information is common knowledge among a group of agents, it means that for every combination of agents in that group, they all know that each other knows that each other knows ... ad infinitum. This is significantly stronger than group knowledge, and it is easy to come up with relational models in which a formula is group knowledge, but not common knowledge. We will use $C_G\varphi$ to symbolize “it is common knowledge among G that φ .”

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Bibliography